Covariant response theory beyond RPA and its application

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Abstract

The covariant particle-vibration coupling model within the time blocking approximation is employed to supplement the Relativistic Random Phase Approximation (RRPA) with coupling to collective vibrations. The Bethe-Salpeter equation in the particle-hole channel with an energy dependent residual particle-hole (p-h) interaction is formulated and solved in the shell-model Dirac basis as well as in the momentum space. The same set of the coupling constants generates the Dirac-Hartree single-particle spectrum, the static part of the residual p-h interaction and the particle-phonon coupling amplitudes. This approach is applied to quantitative description of damping phenomenon in even-even spherical nuclei with closed shells ^{208}Pb and ^{132}Sn. Since the phonon coupling enriches the RRPA spectrum with a multitude of ph⊗phonon states a noticeable fragmentation of giant monopole and dipole resonances is obtained in the examined nuclei. The results are compared with experimental data and with results of the non-relativistic approach.

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I. INTRODUCTION

Recent development of experimental facilities with radioactive nuclear beams has stimulated enhanced efforts on the theoretical side to understand the dynamics of the nuclear many-body problem by microscopic methods. The most successful schemes based on the mean field concept use a phenomenological ansatz incorporating as many symmetries of the system as possible and adjust the parameters of functionals to ground state properties of characteristic nuclei all over the periodic table. Of particular interest are the models based on covariant density functionals [1, 2] because of their Lorentz invariance. A large variety of nuclear phenomena have been described over the years within this kind of models: the equation of state in symmetric nuclear matter, ground state properties of finite spherical and deformed nuclei all over the periodic table [3] from light nuclei [4] to super-heavy elements [5], from the neutron drip line, where halo phenomena are observed [6] to the proton drip line [7] with nuclei unstable against the emission of protons [8]. In the small amplitude limit one obtains the relativistic Random Phase Approximation (RRPA) [9]. This method provides a natural framework to investigate collective and non-collective excitations of *ph*-character. It is successful in particular for the understanding of the position of giant resonances and spin-or/and isospin-excitations as the Gamov Teller Resonance (GTR) or the Isobaric Analog Resonance (IAR). Recently it has been also used for a theoretical interpretation of low-lying dipole [10] and quadrupole [11] excitations.

Of course the density functional theory based on the mean field framework cannot provide an exact treatment of the full nuclear dynamics. It is known to break down already in ideal shell-model nuclei such as ^{208}Pb with closed protons and neutron shells. In self-consistent mean field calculations one finds usually a considerably reduced level density at the Fermi surface as compared with the experiment. The RRPA describes very well positions of giant resonances but underestimates their width considerably. To solve the level density problem the covariant theory of particle-vibration coupling has been developed and applied in the Ref. [12]. In the present work we formulate the covariant response theory employing the particle-vibration coupling model within the time-blocking approximation [13, 14, 15, 16] to describe the spreading of multipole giant resonances in even-even spherical nuclei.

II. FORMALISM

In the relativistic nuclear mean-field theory motion of a single nucleon is described by the Dirac equation with an effective mass m^* and a generalized four-vector of the momentum P^μ:

$$\big(\gamma^\mu P_\mu - m^*\big)|\psi\rangle = 0\,. \tag{1}$$

These quantities are determined by the scalar $\tilde\Sigma_s$ and vector $\tilde\Sigma^\mu = (\tilde\Sigma^0, \tilde{\boldsymbol\Sigma})$ parts of the mass operator (self-energy) $\tilde\Sigma$ within mean-field approximation:

$$m^* = m + \tilde\Sigma_s\,, \qquad P_\mu = p_\mu - \tilde\Sigma_\mu = \Big(i\frac{\partial}{\partial t} - \tilde\Sigma_0, i\nabla + \tilde{\boldsymbol\Sigma}\Big)\,, \tag{2}$$

$\tilde\Sigma_s$ is generated by the scalar σ-meson field. When we go beyond the mean-field approximation, we have to take into account that in the general case the full self-energy Σ is non-local in the space coordinates and also in time. This non-locality means that its Fourier transform has both momentum and energy dependence. Let us decompose the total self-energy matrix into two components, a static local and and an energy dependent non-local term:

$$\Sigma(\mathbf{r},\mathbf{r}';\varepsilon) = \tilde\Sigma(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}') + \Sigma^e(\mathbf{r},\mathbf{r}';\varepsilon), \tag{3}$$

where index "e" indicates the energy dependence. Due to time-reversal symmetry and the absence of currents the space-like components of $\tilde\Sigma$ vanish, therefore only scalar and the time-like components of the mean field are considered in the following.

The quantity $\tilde\Sigma(\mathbf{r})$ is assumed to be the RMF self-energy generated by the σ, ω and ρ-meson fields within the framework of the no-sea approximation (see, for instance, Ref [9]). To describe the non-local part $\Sigma^e(\mathbf{r},\mathbf{r}';\varepsilon)$ we apply the covariant version of the particle-phonon coupling model [12]. Due to the decomposition (3) it is convenient to work in the shell-model Dirac basis $\{|\psi_k\rangle\}$ which diagonalizes the energy-independent part of the Dirac equation:

$$h^{\mathcal{D}}|\psi_k\rangle = \varepsilon_k|\psi_k\rangle, \qquad h^{\mathcal{D}} = \boldsymbol{\alpha}\mathbf{p} + \beta(m+\tilde\Sigma_s) + \tilde\Sigma_0, \tag{4}$$

where $h^{\mathcal{D}}$ denotes the Dirac hamiltonian with the energy-independent mean field. In the case of spherical simmetry the spinor $|\psi_k\rangle$ is characterized by the set of single-particle quantum numbers $k = \{(k), m_k\}, (k) = \{n_k, j_k, \pi_k, t_k\}$ with the radial quantum number n_k, angular momentum quantum numbers j_k, m_k, parity π_k and isospin t_k. In this basis the Dyson equation for the single-particle Green's function can be formulated as follows:

$$\sum_l \big\{(\varepsilon - \varepsilon_k)\delta_{kl} - \Sigma^e_{kl}(\varepsilon)\big\} G_{lk'}(\varepsilon) = \delta_{kk'}. \tag{5}$$

Matrix elements of the energy-dependent part of the mass operator

$$\Sigma^{e}_{kl}(\varepsilon) = \int d^3r d^3r' \; \bar{\psi}_k(\boldsymbol{r})\Sigma^{e}(\boldsymbol{r},\boldsymbol{r'};\varepsilon)\psi_l(\boldsymbol{r'}) \tag{6}$$

are expressed in terms of the particle-phonon coupling model

$$\Sigma^{e}_{kl}(\varepsilon) = \sum_{q,n} \frac{\gamma^{q(\sigma_n)}_{kn}\,\gamma^{q(\sigma_n)*}_{ln}}{\varepsilon - \varepsilon_n - \sigma_n(\Omega^q - i\eta)}, \qquad \gamma^{q(\sigma)}_{kn} = \delta_{\sigma,+1}\gamma^{q}_{kn} + \delta_{\sigma,-1}\gamma^{q*}_{nk} \tag{7}$$

through the phonon vertexes γ^q and their frequencies Ω^q. They are determined by the following relation:

$$\gamma^{q}_{kl} = \sum_{k'l'} V_{kl',lk'}\hat{\rho}^{q}_{k'l'}, \qquad V_{kl',lk'} = \frac{\delta\tilde{\Sigma}_{l'k'}}{\delta\rho_{lk}}. \tag{8}$$

$V_{kl',lk'}$ denotes the matrix element of the residual interaction which is a functional derivative of the relativistic mean field with respect to nuclear density ρ and $\hat{\rho}^q$ is the transition density. Here we use the linearized version of the model which assumes that $\hat{\rho}^q$ is not influenced by the particle-phonon coupling and can be computed within the relativistic RPA. In the present work the residual interaction is generated by the relativistic NL3 Lagrangian [17]. In the Eq. (7) $\sigma_n = +1$ if n is an unoccupied state of p- or α-types and $\sigma_n = -1$ for an occupied n state of h-type, $\eta \to +0$. α denotes states in the Dirac sea with negative energies which arise in the Lehmann expansion of the single-particle Green's function due to the no-sea approximation.

Equation (5) has been solved numerically in the Ref. [12] in the shell-model of Dirac states. A noticeable increase of the single-particle level density near the Fermi surface relative to the pure RMF spectrum is obtained for ^{208}Pb. This improves the agreement of the single-particle level scheme with experimental data considerably. For the four odd mass nuclei surrounding ^{208}Pb the distribution of the single-particle strength has been calculated and compared with experiment as well as with the results obtained within several non-relativistic approaches.

The nuclear dynamics of an even-even nucleus in a weak external field is described by the linear response function which is a solution of the Bethe-Salpeter equation (BSE) in the particle-hole (p-h) channel. In the beginning it is convenient to consider this equation in the time representation. Let us include the time variable into the set of single-particle quantum numbers and use the following number indexation to simplify the expressions: $1 = \{k_1, t_1\}$. In this notation the BSE for the response function R reads:

$$R(14,23) = -\tilde{G}(1,3)\tilde{G}(4,2) + \frac{1}{i}\sum_{5678} \tilde{G}(1,5)\tilde{G}(6,2)W(58,67)R(74,83), \tag{9}$$

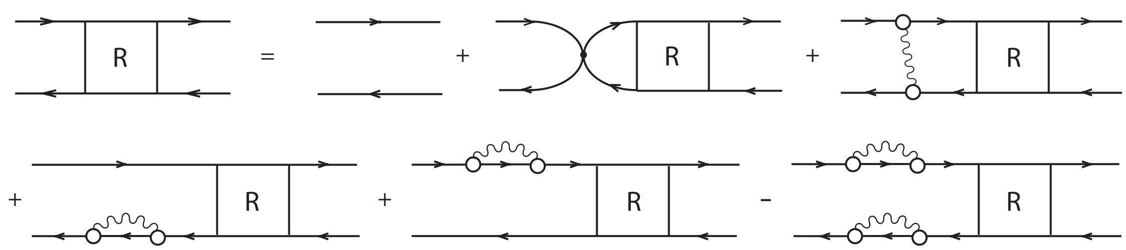

FIG. 1: Bethe-Salpeter equation for the p-h response function in the graphical representation. Solid lines with arrows denote one-body propagators through the particle, hole or antiparticle states, weavy lines denote phonon propagators, empty circles are the particle-phonon coupling amplitudes and the small black circle means the static part of the residual p-h interaction.

where

$$W(14,23) = U(14,23) + i\Sigma^e(1,3)\tilde{G}^{-1}(4,2) + i\tilde{G}^{-1}(1,3)\Sigma^e(4,2) - i\Sigma^e(1,3)\Sigma^e(4,2). \quad (10)$$

Here the summation over number indices implies also integration over respective time variables. $\tilde{G}$ is the mean field single-particle Green's function and U is irreducible in the p-h channel amplitude of the effective interaction including an induced interaction due to the phonon exchange. The graphical representation of the Eq. (9) is shown in Fig. 1.

Using Fourier transformation of the Eq. (9) one comes to an integral equation where both the solution and the kernel are singular with respect to energy variables. Another difficulty arises because the Eq. (9) contains integrations over all time points of the intermediate states. This means that many configurations which are actually more complex than 1p1h⊗phonon are contained in the exact response function. In the Ref. [13] the special time-projection technique was introduced to block the p-h propagation through these complicated intermediate states. It has been shown that for this type of response it is possible to reduce the integral equation to a relatively simple algebraic equation. Obviously, this method can be applied straightforwardly to our case. The full formalism can be found in the Ref. [18]. Making use the above mentioned time projection one can transform the Eq. (9) to the following algebraic equation within the so-called time-blocking approximation:

$$\begin{aligned} R_{k_1k_4,k_2k_3}(\omega) &= \tilde{R}_{k_1k_4,k_2k_3}(\omega) \\ &- \sum_{k_5k_6k_7k_8} \tilde{R}_{k_1k_6,k_2k_5}(\omega)\left[V_{k_5k_8,k_6k_7} + \Phi_{k_5k_8,k_6k_7}(\omega)\right] R_{k_7k_4,k_8k_3}(\omega)\,, \end{aligned} \quad (11)$$

where $\tilde{R}$ is the mean-field p-h propagator, V is the residual interaction defined by the Eq. (8), Φ is the particle-phonon coupling amplitude including phonon contribution both into the self-energy and into the induced interaction. It is supposed that the summation is carried out over the whole Dirac space.

Thus, to describe the observed spectrum of the excited nucleus in the weak external field P within this formalism one needs to solve the Eq. (11) and to calculate the strength function:

$$S(E) = \frac{1}{\pi} \lim_{\Delta \to +0} \mathrm{Im} \sum_{k_1 k_1 k_3 k_4} P^{*}_{k_1 k_2} R_{k_1 k_4, k_2 k_3}(E + i\Delta) P_{k_3 k_4}. \tag{12}$$

The imaginary part Δ of the energy variable is introduced for convenience in order to obtain more smoothed envelope of the spectrum. This parameter has a meaning of an additional artificial width for each excitation. This width emulates effectively contributions from configurations which are not taken into account explicitely.

III. RESULTS AND DISCUSSION

The developed approach is applied to a quantitative description of isoscalar monopole and isovector dipole giant resonances in the even-even spherical nuclei ^{208}Pb and ^{132}Sn. Details of our calculation scheme are given in the Ref. [18]. First, the Dirac equation for single nucleons together with the Klein-Gordon equations for meson fields (RMF problem) are solved simultaneously to obtain the single-particle basis. Second, the RRPA equations [9] are solved to determine for the above mentioned phonons. These two sets form the multitude of ph$\otimes$phonon configurations which enter the particle-phonon coupling amplitude Φ. Third, an equation for density matrix variation (convolution of the Eq. (11) with the external field operator) is solved with this additional amplitude. It provides an enrichment of the calculated spectrum as compared to the pure RRPA. The equation for density matrix variation has been solved both in the momentum and in the Dirac spaces to ensure propriety of our calculational scheme and identical results have been obtained. The calculated strength functions for the isoscalar monopole resonance in ^{208}Pb and ^{132}Sn computed within the RRPA and the RRPA extended by the particle-vibration coupling (RRPA-PC) are given in the Fig. 2. The fragmentation of the resonance caused by the particle-phonon coupling is clearly demonstrated although the spreading width of the monopole resonance is not large because of a strong cancellation between the self-energy diagrams and diagrams with the

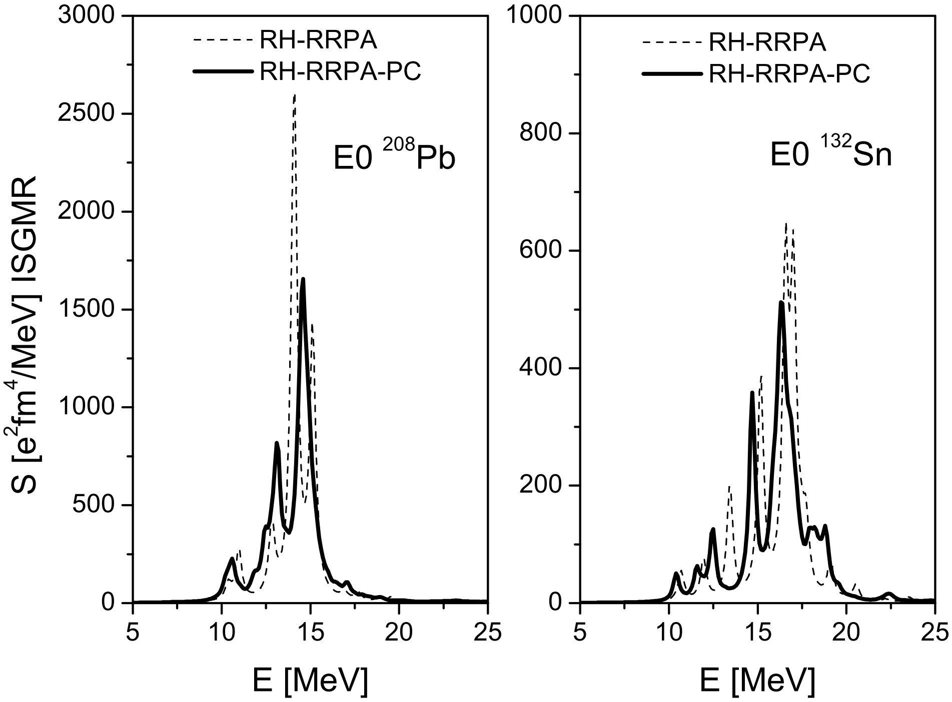

FIG. 2: Isocalar monopole resonance in ^{208}Pb and ^{132}Sn obtained within two approaches: the RRPA (dashed lines) and the RRPA with the particle-phonon coupling RRPA-PC (solid lines). Both calculations are based on the relativistic Hartree (RH) approach with the parameter set NL3.

TABLE I: Lorentz fit parameters of isoscalar E0 strength function in ^{208}Pb and ^{132}Sn calculated within the RRPA and the RRPA extended by the particle-phonon coupling model (RRPA-PC) as compared to experimental data.

		<E> (MeV)	Γ (MeV)
	RRPA	14.13	1.17
^{208}Pb	RRPA-PC	14.02	1.57
	Exp. [19]	13.73(20)	2.58(20)
	RRPA	16.13	1.96
^{132}Sn	RRPA-PC	16.07	2.37

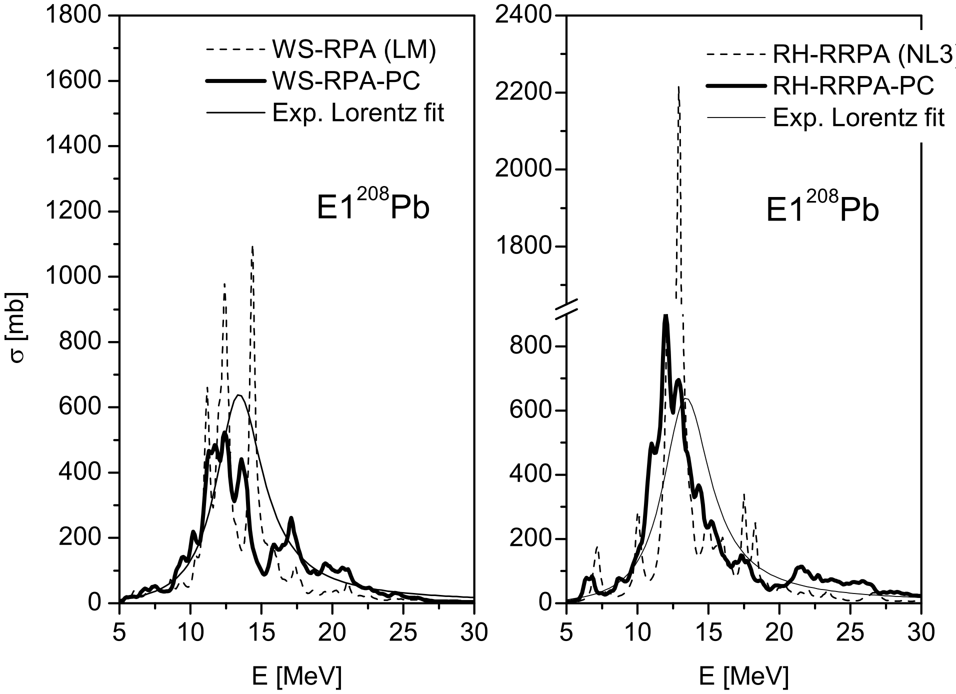

FIG. 3: Isovector E1 resonance in ^{208}Pb: the results obtained within the non-relativistic approach (left panel) with Woods-Saxon (WS) single-particle input and Landau-Migdal (LM) forces and calculations performed within the covariant theory (right panel) based on the relativistic Hartree (RH) approach with the NL3 mean field parameter set. The RPA calculations are shown by the dashed curves, the RPA-PC calculations – by the thick solid curves. Experimental Lorentzian is given by the thin solid curves.

phonon exchange (see Fig. 1). The mean energies and widths of these resonances are presented in the Table I. As experimental data we display the numbers adopted in the Ref. [19] for the calculation of the nuclear matter compressibility from the evaluation of a series of data obtained in different experiments for the isoscalar monopole resonance in ^{208}Pb.
The calculated photoabsorption cross sections for the isovector dipole resonance in ^{208}Pb and ^{132}Sn are given in the Figs. 3 and 4 respectively. The left panels show the results obtained within the non-relativistic semi-phenomenological approach developed in the Ref. [16] which includes particle-phonon coupling on the base of Woods-Saxon single-particle input and Landau-Migdal forces. The right panels show the results of the calculations

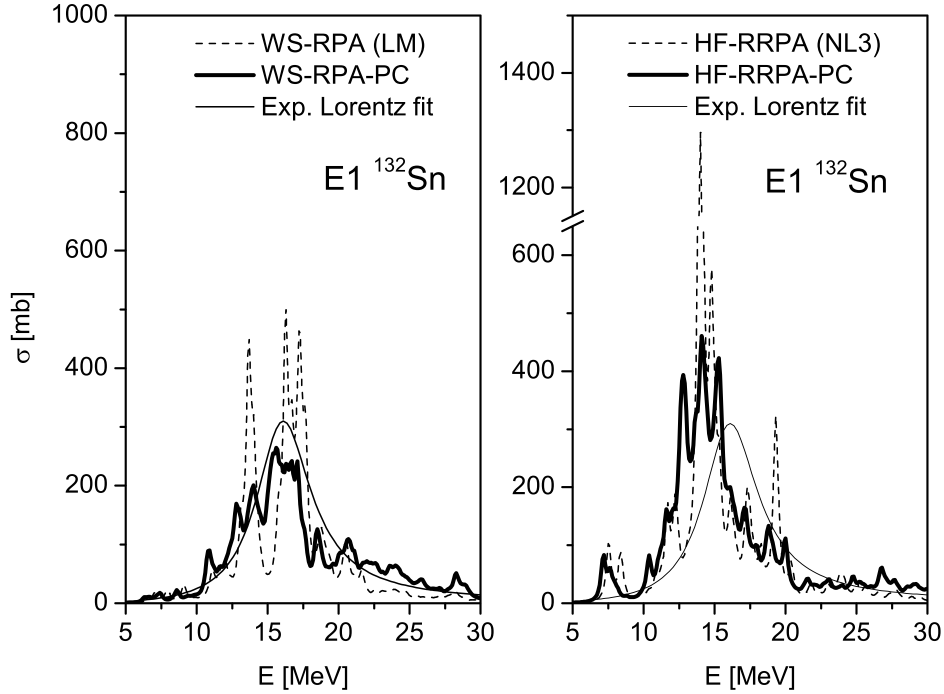

FIG. 4: The same as in Fig. 3 but for ^{132}Sn.

TABLE II: Lorentz fit parameters of the E1 photoabsorption cross section in ^{208}Pb and ^{132}Sn calculated within the RRPA and the RRPA extended by the particle-phonon coupling model (RRPA-PC) as compared to experimental data.

		<E> (MeV)	Γ (MeV)	EWSR (%)
	RRPA	13.1	2.5	120
^{208}Pb	RRPA-PC	12.8	3.8	114
	Exp. [20]	13.4	4.1	
	RRPA	14.7	3.0	112
^{132}Sn	RRPA-PC	14.3	3.8	108
	Exp. [21]	16.1(7)	4.7(2.1)	

within the relativistic approach developed in the present work. To make the comparison reasonable, calculations within the non-relativistic framework have been performed with box boundary conditions for the Schrödinger equation in the coordinate representation which ensures completeness of the single-particle basis. In both cases one can see the noticeable fragmentation of the resonances due to the particle-phonon coupling. Moreover, one can find more or less the same level of agreement with experimental data for these two calculations. In case of the isovector E1 resonance in ^{132}Sn this is, however, not so clear because the integral characteristics of the resonance obtained in the experiment of tthe Ref. [21] are given with relatively large discrepancies. But the difference which is of fundamental importance is that in the semi-phenomenological approach one usually fits the parameters on all three stages of the calculation: first, the Woods-Saxon well depth is varied to obtain single-particle levels equal to experimental values, second, one of the Landau-Migdal force parameters is adjusted to get phonon energies at the experimental positions (for each mode) and, third, another Landau-Migdal force parameter is varied to reproduce the centroid of the giant resonance. Although the varying of the parameters is performed in relatively narrow limits, in some cases it is necessary to obtain realistic results. In contrast, within the fully covariant microscopic approach developed in the present work no adjustment of parameters is made.

The mean energies and widths of the isovector E1 resonance computed within the RRPA and the RRPA-PC are displayed in the Table II. For the isovector E1 resonance as well as for the isoscalar E0 resonance Lorentz fits have been made according to the method developed in the Ref. [22] within the energy region between one and three neutron separation energies. For the E1 resonance in ^{132}Sn the lower energy limit is slightly higher in order to separate the distinct group of pygmy states from the giant resonance.

IV. SUMMARY

The relativistic random phase approximation is extended by the particle-vibrational coupling model. The Bethe-Salpeter equation is formulated in the two-body basis of Dirac states. Amplitude of the effective interaction entering this equation contains the static part originating from the pure relativistic mean field as well as the energy-dependent part caused by the particle-vibrational coupling. The latter term has been considered within the time-blocking approximation in a fully relativistic way using the covariant form of the nucleon

mass operator.

The developed approach is applied to the computation of spectroscopic characteristics of nuclear excited states in a wide energy range up to 30 MeV for even-even spherical nuclei. An equation for the density matrix variation is solved in the Dirac space as well as in the momentum space. The particle-phonon coupling amplitudes of collective vibrational modes below the neutron separation energy have been computed within the self-consistent RRPA using the parameter set NL3 for the Lagrangian. The same force has been employed in giant resonance calculations for the static part of the effective p-h interaction. Therefore a fully consistent description of giant resonances is performed.

Noticeable fragmentation of the isoscalar monopole and isovector dipole giant resonances in ^{208}Pb and ^{132}Sn is obtained due to the particle-vibrational coupling. This leads to the appearance of a significant spreading width as compared to RRPA calculations. This is in agreement with experimental data as well as with the results obtained within the non-relativistic approaches [16, 23].

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